APPENDIX B: GENERATION OF EQUIVALENT WALL

Generation of Equivalent Wall

There are several ways the equivalent wall technique may generate a one-dimensional *n*-layer structure with similar dynamic thermal properties as the actual wall. In general, the number of layers should be three or more. Three-layer structures are the simplest way of achieving a model that gives good results. With a two-layer structure, values of the structure factor φ_{ie} above 3/16 may not be reproduced.

Equations (A26) through (A28) are not all independent, because of the identity (A25). Notice that they are linear in C_n and quadratic in R_n. Summing up (A26) with (A27), or (A27) with (A28), yields equations linear in R_n. As the basis of an equivalent wall model one may use the following system of independent equations:

$$\varphi_{ii} + \varphi_{ie} = \frac{1}{RC}\sum_{m=1}^{n} C_m \left[\frac{R_m}{2} + R_{m-e} \right] \tag{A37}$$

$$\varphi_{ie} = \frac{1}{R^2 C}\sum_{m=1}^{n} C_m \left[-\frac{R_m^2}{3} + \frac{R_m R}{2} + R_{i-m} R_{m-e} \right] \tag{A38}$$

$$R = \sum_{m=1}^{n} R_m \tag{A39}$$

$$C = \sum_{m=1}^{n} C_m \tag{A40}$$

The response factors can be reproduced using any general-purpose numerical code, such as HEATING 7.2. With a set of response factors for the wall assembly, the determination of structure factors follows from the equations (A37) and (A38). A spreadsheet has been developed to calculate thermal structure factors, using the equations presented in this report. The user must enter the series of response factors, as generated from any dynamic 3-D program, along with the corresponding number of response factors. The spreadsheet calculates the product of these two values. Once all the response factors and numbers have been entered, the user must divide the summation of these products by the overall wall capacity. A copy of the spreadsheet program is included with this report.

Equations (A37) through (A40) represent four conditions, to be satisfied by 2*n* variables. Thus, some of them must be treated as “parameters”, which means that their values must be initially given, and then the remaining “variables” are to be determined solving

the equations. Assuming n = 3, and given R, C, φ_{ii}, φ_{ie} and a set of first approximations for R_1, R_2, and R_3, one may easily solve the system of linear equations (A37), (A38) and (A40) for C_1, C_2, and C_3. However, the solution obtained in this way may not be correct, in the sense that some C_n may be negative. It is thus necessary to generate, randomly or with some logic, many combinations of R_n values, to find admissible combinations of C_n values.

Thermophysical properties of the layers may then be established, if necessary, to match R_n and C_n values and total thickness of the wall.

For n = 3, the procedure would this may look like this:

- Assume q_{max}, p_{max} – as numbers in the range (5, 100).
- Let q run from 1 to q_{max}-1 and p run from 1 to p_{max}-1, respectively.
- $R_1 = R \cdot q/q_{max}$, $R_2 = (R - R_1) \cdot p/p_{max}$, $R_3 = R - R_1 - R_2$
- For each q and p solve Equations (A37), (A38) and (A40) for C_1, C_2, C_3, assuming they are all positive.

The algorithm described above has been included in an “equivalent wall generator”; developed as a stand-alone computer program to provide the thermophysical properties for the equivalent wall layers. This program is included as part of this report.

Another way is to generate first a set of capacitances C_n and then seek for the resistances R_n, to satisfy Equations (A37), (A38) and (A39). For n > 3 more R_n and C_n values must be treated as “parameters”; they may be either generated or fixed.

When fixing some values, one should take into account that material configuration of the equivalent wall. This is described for example, as “thermal mass inside” or “material of low conductivity” outside. The configuration should be similar to that of the original 3-D wall. One may obtain in this way many equivalent wall models. The equivalent wall model is not unique, even for given n, however different models lead, in general, to very similar dynamic thermal properties. One may then examine generated models to choose the best one.

The next step is to calculate response factors and compare them with 3-D response factors – using finite differences or the Laplace transform method [see Kusuda, Stephenson and Mitalas, Clarke]. Elements of the s-transfer matrix depend on the argument $l_n\sqrt{s/\alpha_n}$, where s is the Laplace transform variable, l_n is thickness and α_n thermal diffusivity of the homogeneous n-th layer. This argument may be represented as

$\sqrt{s R_n C_n}$. Therefore the set of R_n and C_n values is sufficient to calculate response factors and z-transfer function coefficients. While the response factors and z-transfer function coefficients are useful for comparison, they are not required by professional programs that use input in the form of thermophysical parameters of subsequent layers.

Some equivalent wall models, for given thickness of the layers and specific heat, may produce values of conductivity and density which are outside of the range accepted for building materials. While this effect is of no importance for the response factors calculations, which depend on resistance and capacitance only, it should be avoided when using whole building energy simulations.

A summary of the equivalent wall generation method is presented in Figure 1.

Calculation of Thermal Structure Factors
- Spreadsheet

Thermal structure factors are determined from Equations (14), (15) and (A25), given the response factors and total capacitance of the wall generated from the 3-D model.

$$\delta \sum_{n=1}^{\infty} n X_n = -C \varphi_{ii} \tag{14}$$

$$\delta \sum_{n=1}^{\infty} n Y_n = C \varphi_{ie} \tag{15}$$

$$\varphi_{ii} + 2\varphi_{ie} + \varphi_{ee} = 1 \tag{A25}$$

Generation of Equivalent Wall (Thermophysical Properties of Equivalent Wall Layers)
- 'Equivalent Wall Generator'

(1) Assume some number of equivalent layers for the wall structure. In general, three-layer structures are the simplest way of achieving a model that gives good results.

(2) Generate, randomly or with some logic, a set of resistances R_n (or capacitances C_n) for each layer, then seek the capacitances C_n (or resistances R_n) to satisfy Equations (A37), (A38) and (A40). The thermal structure factors and overall R-value must match those for the 3-D wall assembly.

$$\varphi_{ii} + \varphi_{ie} = \frac{1}{RC} \sum_{m=1}^{n} C_m \left[\frac{R_m}{2} + R_{m-e} \right] \tag{A37}$$

$$\varphi_{ie} = \frac{1}{R^2 C} \sum_{m=1}^{n} C_m \left[-\frac{R_m^2}{3} + \frac{R_m R}{2} + R_{i-m} R_{m-e} \right] \tag{A38}$$

$$R = \sum_{m=1}^{n} R_m \tag{A39}$$

$$C = \sum_{m=1}^{n} C_m \tag{A40}$$

Combinations of R_n values can be generated using the following procedure, to find admissible combinations of C_n values.

- Assume q_{max}, p_{max} – as numbers in the range (5, 100).
- Let q run from 1 to q_{max}-1 and p run from 1 to p_{max}-1, respectively.
- $R_1 = R \cdot q/q_{max}$, $R_2 = (R - R_1) \cdot p/p_{max}$, $R_3 = R - R_1 - R_2$
- For each q and p solve Equations (A37), (A38) and (A40) for C_1, C_2, C_3, assuming they are all positive.

Calculation of Equivalent Wall Response Factors and Z-Transfer Function Coeff.
- Laplace Transform Method

Given that the equivalent wall is a one-dimensional model, response factors and z-transfer function coefficients are calculated using the standard, Laplace transform method, [Kusuda, Stephenson and Mitalas, Clarke].

Comparison

The equivalent wall model is an iterative procedure. By comparing the response factors and z-transfer function coefficients for the equivalent wall and 3-D models, one can adjust the number and capacitances of equivalent wall layers so that the equivalent wall model can generate results that more closely resemble those of the 3-D model.

Figure 1: Equivalent Wall Generation Summary