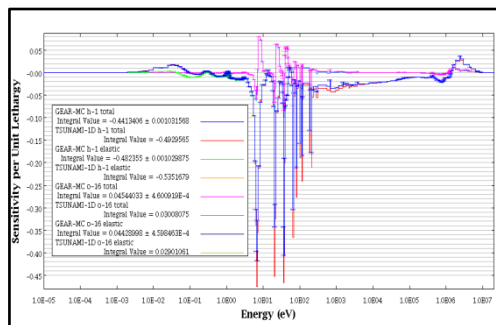
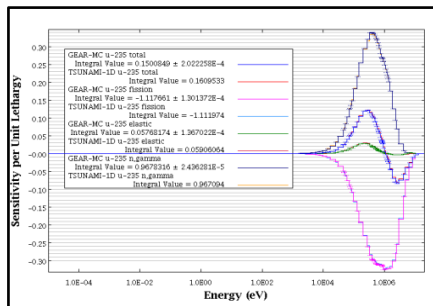


Continuous-Energy TSUNAMI Sensitivity Capabilities in SCALE 6.2



$$S_{R,\Sigma} = \frac{\delta R/R}{\delta \Sigma/\Sigma}$$



Dr. Christopher M. Perfetti
Radiation Transport Group
Reactor and Nuclear Systems Division
Oak Ridge National Laboratory

Overview

- The TSUNAMI (Tools for Sensitivity and Uncertainty Analysis Methodology Implementation) capabilities within the SCALE code system make use of sensitivity coefficients for an extensive number of criticality safety applications, including:
 - Quantifying the data-induced uncertainty in the eigenvalue of critical systems,
 - Assessing the neutronic similarity between different systems,
 - Quantifying computational biases, and
 - Guiding nuclear data adjustment studies.
- This presentation will provide a brief overview of the sensitivity analysis methods in SCALE 6.2 in CE TSUNAMI-3D.

Calculating Sensitivity Coefficients

Relative sensitivity of k_{eff} to single energy group of a particular nuclide-reaction pair cross section, $S_{x,g}$, is expressed as:

$$S_{k, \Sigma_{x,g}} = \frac{\partial k_{eff} / k_{eff}}{\partial \Sigma_{x,g} / \Sigma_{x,g}}$$

$$S_{k, \Sigma(\vec{r})} \equiv \frac{\partial k / k}{\partial \Sigma(\vec{r}) / \Sigma(\vec{r})} = - \frac{\Sigma(\vec{r})}{k} \frac{\left\langle \phi^\dagger(\vec{\xi}) \left(\frac{\partial A[\Sigma(\vec{\xi})]}{\partial \Sigma(\vec{r})} - \frac{1}{k} \frac{\partial B[\Sigma(\vec{\xi})]}{\partial \Sigma(\vec{r})} \right) \phi(\vec{\xi}) \right\rangle}{\left\langle \phi^\dagger(\vec{\xi}) \frac{1}{k^2} B[\Sigma(\vec{\xi})] \phi(\vec{\xi}) \right\rangle}$$

where

ϕ = neutron flux;

ϕ^\dagger = adjoint neutron flux

$k = k_{eff}$, the largest of the eigenvalues

A = operator that represents all of the transport equation except for the fission term

B = operator that represents the fission term of the transport equation

Σ = problem-dependent resonance self-shield macroscopic cross sections

$\vec{\xi}$ = phase space vector; and

$\langle \rangle$ indicate integration over space, direction and energy variables.

Calculating Sensitivity Coefficients

- For a sample capture reaction (*cap.*), the First-Order Perturbation Equation reduces to something like:

$$S_{k, \Sigma_{cap.}} = \frac{\delta k / k}{\delta \Sigma_{cap.} / \Sigma_{cap.}} = \frac{\langle \Phi^\dagger \Sigma_{cap.} \Phi \rangle}{\frac{1}{k} \langle \Phi^\dagger \Sigma_{fis.} \Phi \rangle}$$

- Tallying reaction rates is relatively straightforward for a Monte Carlo code.
- The challenge is therefore tallying the forward and adjoint fluxes as a function of space, energy, and angle.

Calculating Sensitivity Coefficients

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- The challenge is therefore tallying the forward and adjoint fluxes as a function of space, energy, and angle.

CE TSUNAMI-3D Sensitivity Methods

Eigenvalue Sensitivity Calculations

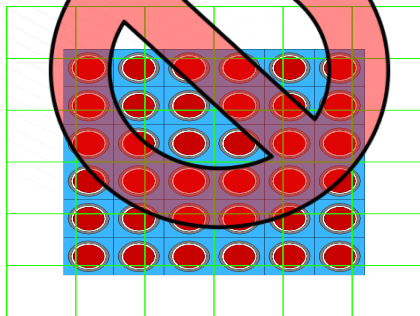
- CLUTCH Method (cet=1)
- IFP Method (cet=2)

Generalized Perturbation Theory Sensitivities

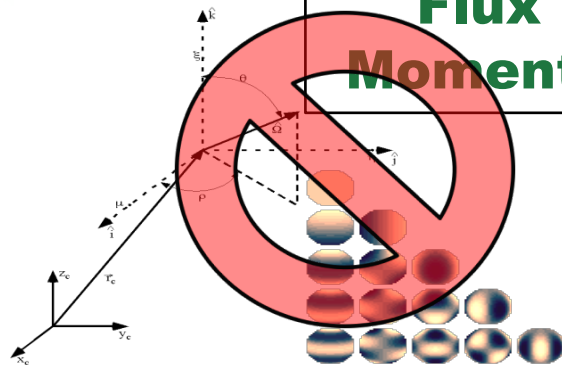
- GEAR-MC Method: CLUTCH only (cet=4)
- GEAR-MC Method: CLUTCH + IFP (cet=5)

Things you need for a multigroup TSUNAMI-3D Calculation:

Spatial Flux Mesh

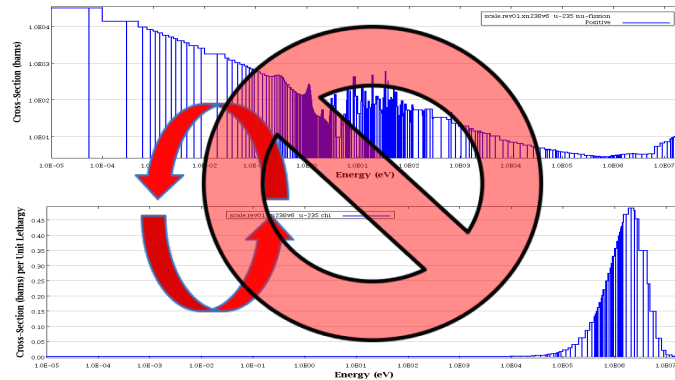


Flux Moments



$$\tilde{\phi}_j = \int \phi(\hat{\Omega}) R_j(\hat{\Omega}) d\hat{\Omega}$$

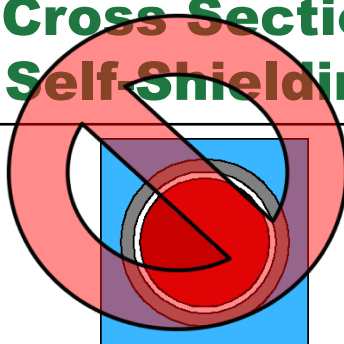
Separate Adjoint Transport Solution



Volume Calculations

$$\text{points} = \frac{\text{volume}}{\ln(\text{volume}/5000)}$$

Cross Section Self-Shielding



Expert Judgment

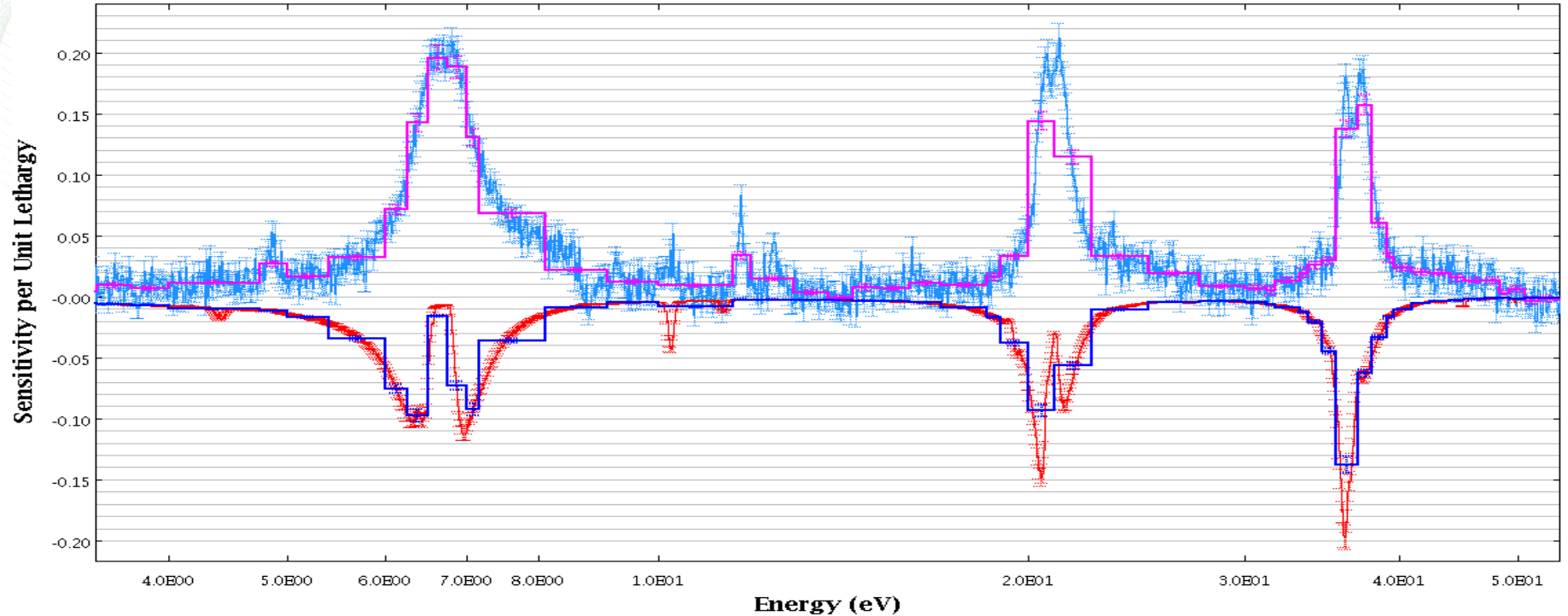


Why use Continuous Energy?

- CE TSUNAMI-3D uses cutting-edge Monte Carlo methods to calculate sensitivity coefficients, and requires:
 - No flux moment calculations
 - No spatial flux mesh (sort of)
 - No volume calculations
 - No problem-dependent cross section self-shielding
 - No implicit sensitivity effects
 - No adjoint transport simulation, just one forward simulation
- CE TSUNAMI-3D avoids the large memory footprints that can be required by multigroup TSUNAMI-3D.
- Use of continuous-energy physics more accurately models the physics of neutron interactions (see: the *read energy* input block).

H-1 Elastic Scatter Sensitivity
238-group CLUTCH VS
Microgroup CLUTCH

U-238 Capture Sensitivity
238-group CLUTCH VS
Microgroup CLUTCH



Why NOT use Continuous Energy?

- The simulation runtimes are usually longer than for multigroup TSUNAMI-3D.
- In many applications multigroup TSUNAMI-3D calculations already provide sufficient accuracy.
- Some problems may still require a spatial flux mesh, significant computational memory, and/or expert judgment.

CE TSUNAMI-3D Sensitivity Methods

Eigenvalue Sensitivity Calculations

- CLUTCH Method (cet=1)
- **IFP Method (cet=2)**

Generalized Perturbation Theory Sensitivities

- GEAR-MC Method: CLUTCH only (cet=4)
- GEAR-MC Method: CLUTCH + IFP (cet=5)

Iterated Fission Probability Method

- The Iterated Fission Probability (IFP) method calculates adjoint-weighted tallies using the notion that the importance of an event is proportional to the population of neutrons present in the system during some future generation.
- In practice, the IFP method can require storing reaction rate tallies for a significant number of generations.
- In CE TSUNAMI-3D, the IFP method is used by setting: **cet=2**
- The number of “latent generations” is set using the **cfp=#** parameter.

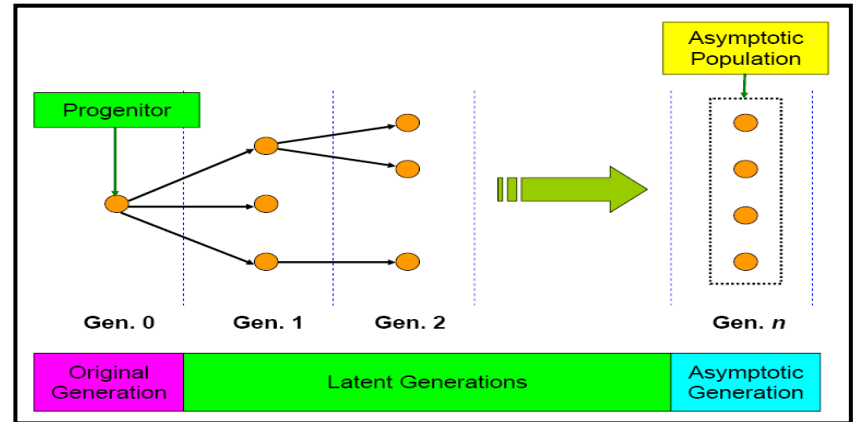


Illustration of the IFP process. Image courtesy of Brian Kiedrowski.

How Many Latent Generations Do We Need?

- IFP calculations should use somewhere between 2 and 20 latent generations to obtain accurate sensitivity tallies.
 - In practice, most simulations require between 5 and 10 latent generations.
- The memory footprint of SCALE IFP calculations scales linearly with the number of latent generations.
 - Users should use enough latent generations to obtain accurate sensitivity coefficients, but also as few as possible to minimize the simulation's memory footprint.

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Let's Try CE TSUNAMI-3D

```
SCALE
File Edit View Run Help
Reload godiva_example.inp Save godiva_example.inp Save godiva_example.inp as Close godiva_example.inp Print Cut Copy Paste Undo Redo Find

Navigation
godiva_example.inp
Filter
godiva_example.inp

document SCALE 6.2 Run
1 =csas5
2 godiva-k5
3 ce_v7_endf
4 read composition
5 u-234 1 0 0.000491995 300 end
6 u-235 1 0 0.0449996 300 end
7 u-238 1 0 0.002498 300 end
8 end composition
9 read geometry
10 global unit 1
11 sphere 1 1 8.741
12 end geometry
13 end data
14 end
15

Line: 15, Col: 1 Validation Messages
```

Let's Try CE TSUNAMI-3D

```
SCALE
File Edit View Run Help
Reload godiva_example_IFP.inp Save godiva_example_IFP.inp as Close godiva_example_IFP.inp Print Cut Copy Paste
Navigation
godiva_example_IFP.inp*
document SCALE 6.2 Run
Filter
godiva_example_IFP.inp*
1 =tsunami-3d-k5
2 godiva-k5
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9 read parameter
10 cet=2
11 cfp=5
12 end parameter
13 read geometry
14 global unit 1
15 sphere 1 1 8.741
16 end geometry
17 end data
18 end
19
```

IFP Method Memory Requirements

- The IFP method allows for very accurate sensitivity coefficient calculations, but sometimes encounters large computational memory footprints and long problem runtimes.
- For a model of a typical PWR with depletion isotopics...

38,000 *unique isotope-regions*
× **12** *reactions per isotope*
× **44** *energy groups*
× **11** *generations of storage*
× **10,000** *particles per generation*
× **8** *bytes per double*
= **17,656** *gigabytes of memory*

CE TSUNAMI-3D Sensitivity Methods

Eigenvalue Sensitivity Calculations

- **CLUTCH Method** (cet=1)
- IFP Method (cet=2)

Generalized Perturbation Theory Sensitivities

- GEAR-MC Method: CLUTCH only (cet=4)
- GEAR-MC Method: CLUTCH + IFP (cet=5)

CLUTCH/Contributon Methodology

- The CLUTCH method calculates the importance of collisions by tallying how many fission neutrons are created by a particle after it leaves the collision:

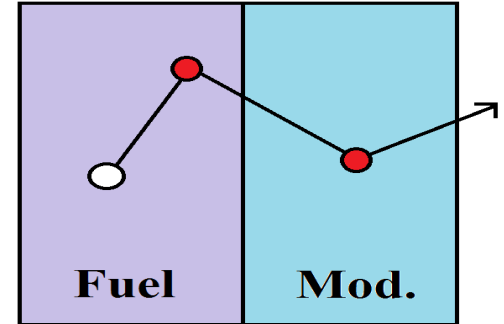
$$\phi^\dagger(\tau_s) = \int_V G(\tau_s \rightarrow r) F^\dagger(r) dr,$$

...where:

$G(\tau_s \rightarrow r)$ = The number of fission neutrons created at r by the neutron originating in the phase space τ_s .

$F^*(r)$ = The average importance of fission neutrons born at r , or:

$$F^\dagger(r) = \int_E \int_\Omega \frac{\chi(r, E)}{4\pi} \phi^\dagger(r, E, \Omega) d\Omega dE.$$



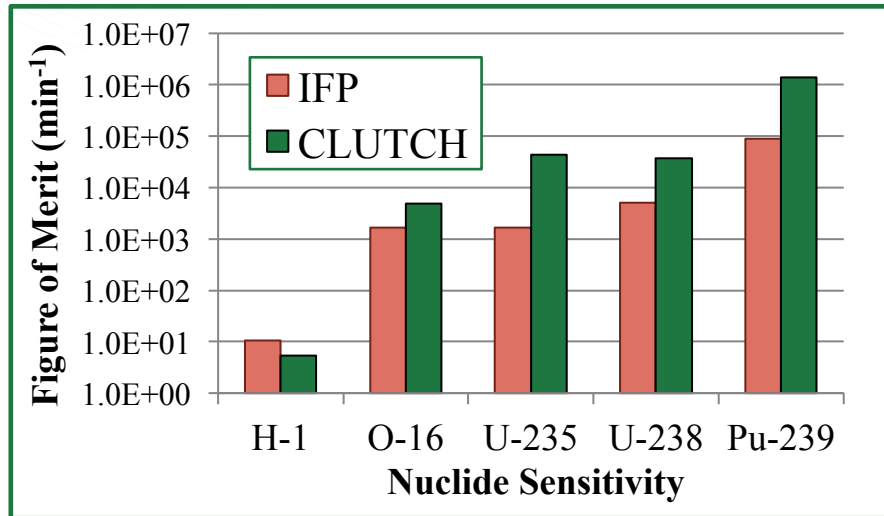
CLUTCH Memory Requirements

- The CLUTCH method requires one to store reaction rate tallies for every collision a particle sees from birth until death.
- Unlike the IFP method, this information is freed after a particle dies and is not carried for multiple generations.

38,000 *unique isotope-regions*
× **12** *reactions per isotope*
× **4,000** *collisions per history*
× **8** *bytes per double*
= **14.6** *gigabytes of memory*

CLUTCH VS IFP

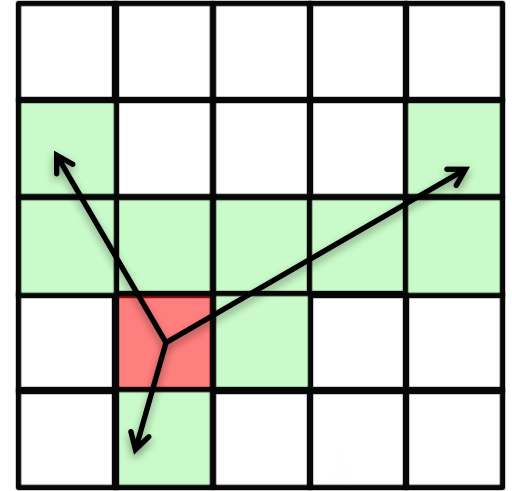
- The CLUTCH method is more efficient than IFP (both in terms of speed and memory usage).
- The downside to CLUTCH is that you need to compute $F^*(r)$.



Sensitivity Method Memory Usage			
Model	IFP	CLUTCH	Memory Reduction Factor
Fuel Pin	2,113 MB	1.06 MB	1,990
Godiva	26 MB	0.12 MB	220
HMF-025-005	1,675 MB	0.16 MB	10,470
LCT-010-014	19,509 MB	25 MB	780
NAC-UMS	21,201 MB	3,416 MB	6.2

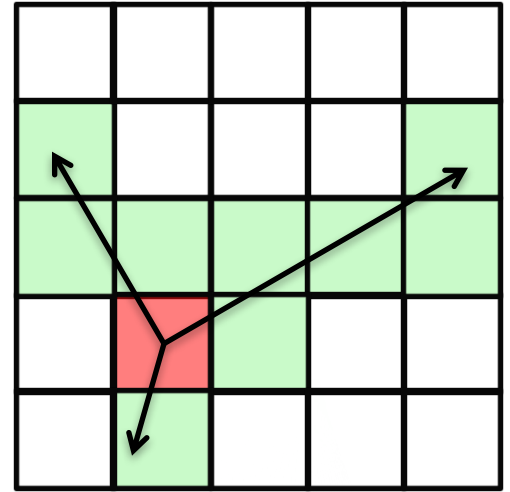
The $F^*(r)$ Function

- The CLUTCH Method uses an importance weighting function, $F^*(r)$, to compute multi-generational sensitivity effects.
- The $F^*(r)$ function describes the average response importance generated by fission neutrons born at location r .
- The $F^*(r)$ function can be calculated using the IFP method during inactive generations with no significant loss of accuracy and with significant memory savings.



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- The $F^*(r)$ function can be calculated using the IFP method during inactive generations with no significant loss of accuracy and with significant memory savings.



How to use CLUTCH with an $F^*(r)$ Mesh

- Set **cet=1** to enable CLUTCH.
- Set **cfp=#** to set the number of latent generations for the IFP calculation that populates the $F^*(r)$ mesh.
- Consider increasing the number of inactive generations to allow the $F^*(r)$ mesh to converge.
- Set **cgd=#** to tell CE TSUNAMI-3D the ID of the GridGeometry mesh for $F^*(r)$.
- Make the GridGeometry mesh for $F^*(r)$.

Let's Try CE TSUNAMI-3D...with CLUTCH!

```
1 =tsunami-3d-k5
2 godiva-k5
3 ce_v7_endf
4 read composition
5 u-234      1 0 0.000491995 300  end
6 u-235      1 0 0.0449996 300  end
7 u-238      1 0 0.002498 300  end
8 end composition
9 read parameter
10 cet=1
11 cfp=5
12 cgd=10
13 end parameter
14 read geometry
15 global unit 1
16 sphere 1 1 8.741
17 end geometry
18 read GridGeometry 10
19 xlinear 19 -9 9
20 ylinear 19 -9 9
21 zlinear 19 -9 9
22 end GridGeometry
23 end data
24 end
25
```

Note: a 1cm-2cm mesh is sufficiently resolved for most CLUTCH $F^*(r)$ calculations.

Let's Compare our IFP and CLUTCH Sensitivities

Nuclide	IFP Sensitivity	CLUTCH Sensitivity	Difference (# Standard Dev.)
U-234	6.92E-03 ± 6.71E-04	6.37E-03 ± 2.68E-04	-0.76
U-235	8.09E-01 ± 5.18E-03	7.89E-01 ± 1.97E-03	-3.67
U-238	1.69E-02 ± 1.50E-03	1.61E-02 ± 5.43E-04	-0.46

Best Practices for $F^*(r)$ Mesh Generation

- An $F^*(r)$ mesh with 1cm – 2cm mesh intervals is generally sufficiently resolved to generate accurate sensitivity coefficients.
- The $F^*(r)$ mesh must only cover all fissionable regions in a problem.
- Setting **cfp=-1** will run CLUTCH assuming that $F^*(r)=1$ everywhere.
 - Useful for models of infinitely-reflected systems.
- Since the $F^*(r)$ mesh is generated during skipped generations, NSK should be adjusted so that the $F^*(r)$ tallies can converge.
 - In general, simulating between 1 and 100 inactive particle histories per $F^*(r)$ mesh interval will produce an accurate $F^*(r)$ tally.
 - Our Godiva problem used a mesh with 5,832 intervals ($18 \times 18 \times 18$);
5,832 mesh intervals \times 100 histories per interval / 1,000 particles per gen. =
~500 skipped generations.

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Improving the CLUTCH Input

```
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5 u-234      1 0 0.000491995 300 end
6 u-235      1 0 0.0449996 300 end
7 u-238      1 0 0.002498 300 end
8 end composition
9 read parameter
10 cet=1
11 cfp=5
12 cgd=10
13 nsk=500
14 gen=1000
15 fst=yes
16 end parameter
17 read geometry
18 global unit 1
19 sphere 1 1 8.741
20 end geometry
21 read GridGeometry 10
22 xlinear 19 -9 9
23 ylinear 19 -9 9
24 zlinear 19 -9 9
25 end GridGeometry
26 end data
27 end
28
```

Best Practices for $F^*(r)$ Mesh Generation

- Setting the FST=yes parameter will produce a .3dmap file showing the $F^*(r)$ mesh that was calculated.
- At the end of the inactive generations, SCALE will summarize the convergence of your $F^*(r)$ mesh in a warning message.

```
499      1.03074E+00      1.00056E+00      1.05296E-03      6.11893E+00      6.45500E-01
500      1.05307E+00      1.00067E+00      1.05612E-03      6.09369E+00      6.46667E-01

F*(r) Convergence Statistics:
WARNING: Of the      3682 F*(r) mesh intervals that scored tallies...
  99.19% of the F*(r) tallies contain more than 5% uncertainty;
  61.46% of the F*(r) tallies contain more than 10% uncertainty;
  23.76% of the F*(r) tallies contain more than 20% uncertainty; and
   5.38% of the F*(r) tallies contain more than 50% uncertainty.

501      9.82646E-01      1.00063E+00      1.05461E-03      6.04676E+00      6.49000E-01
502      9.64969E-01      1.00056E+00      1.05492E-03      6.17351E+00      6.51167E-01
```


Updated Sensitivity Coefficients

Nuclide	IFP Sensitivity	CLUTCH Sensitivity	Improved CLUTCH Run
U-234	6.92E-03 ± 6.71E-04	6.37E-03 ± 2.68E-04 (-0.76 σ)	7.73E-03 ± 2.71E-04 (1.13 σ)
U-235	8.09E-01 ± 5.18E-03	7.89E-01 ± 1.97E-03 (-3.67 σ)	8.01E-01 ± 1.81E-03 (-1.46 σ)
U-238	1.69E-02 ± 1.50E-03	1.61E-02 ± 5.43E-04 (-0.46 σ)	1.80E-02 ± 5.68E-04 (0.71 σ)

TSUNAMI-3D Sensitivity Method Summary

	Multigroup TSUNAMI	IFP	CLUTCH
Accuracy	Good	Excellent	Excellent
Speed	Good	Good	Excellent
Efficiency	Excellent	Good	Excellent
Memory Requirements	Limiting	Limiting	Typically Fine
Ease of Use	Requires a Flux Mesh	Very Easy	Must Calculate $F^*(r)$

CE TSUNAMI-3D Sensitivity Methods

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Generalized Perturbation Theory Sensitivities

- GEAR-MC Method: CLUTCH only (cet=4)
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Generalized Perturbation Theory

- Generalized Perturbation Theory (GPT) estimates sensitivity coefficients for any system response that can be expressed as the ratio of reaction rates.

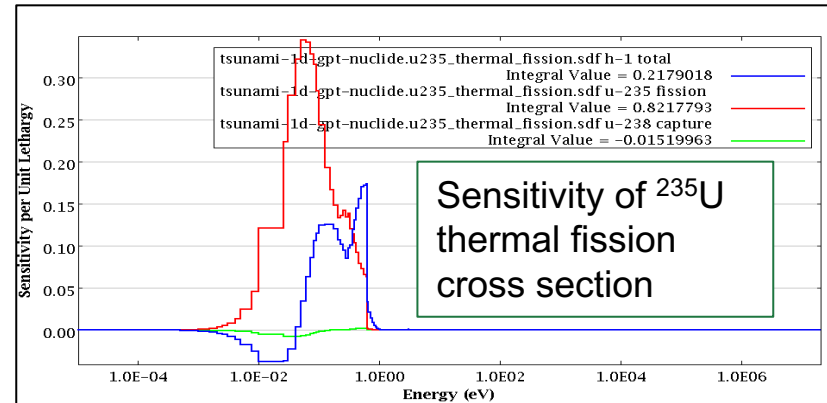
$$S_{R,\Sigma} = \frac{\delta R/R}{\delta \Sigma/\Sigma} \quad R = \frac{\langle \Sigma_1 \phi \rangle}{\langle \Sigma_2 \phi \rangle}$$

- Calculating generalized sensitivity coefficients requires solving an inhomogeneous, or generalized, adjoint equation:

$$L^\dagger \Gamma^\dagger = \lambda P^\dagger \Gamma^\dagger + S^\dagger$$

$$S^\dagger = \frac{1}{R} \frac{\partial R}{\partial \phi} = \frac{\Sigma_1}{\langle \Sigma_1 \phi \rangle} - \frac{\Sigma_2}{\langle \Sigma_2 \phi \rangle}$$

- TSUNAMI offers several tools for performing GPT sensitivity analysis:
 - TSUNAMI-1D: Multigroup analysis using the XSDRN code.
 - TSUNAMI-2D: Multigroup analysis using the NEWT code.
 - TSUNAMI-3D: Continuous-energy analysis using the KENO-Va/VI codes.



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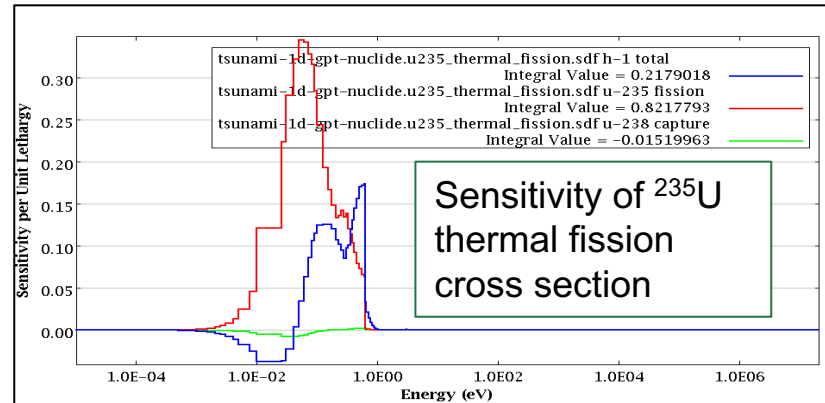
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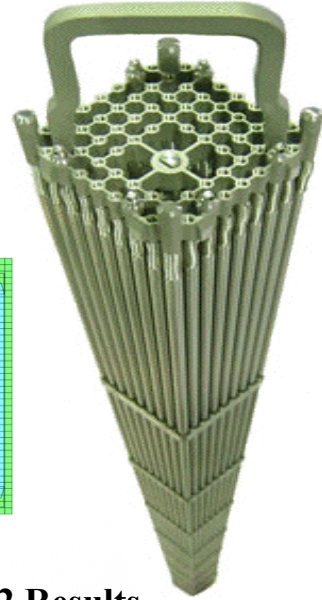
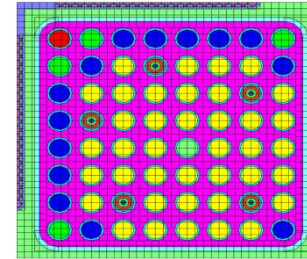
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Generalized Perturbation Theory

- GPT sensitivities can be used to understand the sources and impact of nuclear data uncertainty in responses such as:
 - Relative powers
 - Isotope Conversion ratios
 - Multigroup cross sections
 - Fission ratios
 - Example: $^{239}\text{Pu}(n,f)/^{235}\text{U}(n,f)$
 - Experimental parameters
 - Example: ^{28}p
(ratio of epithermal/thermal ^{238}U capture rates in irradiation foils)



OECD UAM GPT Benchmark Phase 1-2 Results

NUMBER	EXPERIMENT	Type	Format	Value	Xsec Uncert
1	k_infinity	keff	Relative	1.1083E+0	4.98551E-1 % dk/k
2	fission_grp_1	gpt	Relative	1.9155E-3	6.91925E-1 % dR/R
3	fission_grp_2	gpt	Relative	2.7748E-2	3.23440E-1 % dR/R
4	absorpt_grp_1	gpt	Relative	7.1637E-3	8.36728E-1 % dR/R
5	absorpt_grp_2	gpt	Relative	5.3702E-2	2.38082E-1 % dR/R
6	cornerrod_fpf	gpt	Relative	1.1458E+0	1.67147E-1 % dR/R

CE TSUNAMI-3D GPT Response Extension

Original GPT Responses

- Total cross section (MT = 1)
- Fission cross section (MT = 18)
- (n,γ) abs. cross section (MT = 102)
- Neutron prod. cross section (MT = 1452)
- Neutron flux

CE TSUNAMI-3D GPT Response Extension

Original GPT Responses

- Total cross section (MT = 1)
- Fission cross section (MT = 18)
- (n,γ) abs. cross section (MT = 102)
- Neutron prod. cross section (MT = 1452)
- Neutron flux

Updated GPT Responses

- Total cross section (MT = 1)
- Total scatter cross section (MT = 0)
- Elastic scatter cross section (MT = 2)
- Inelastic scatter cross section (MT = 4)
- $(n,2n)$ scatter cross section (MT = 16)
- Fission cross section (MT = 18)
- Total absorption cross section (MT = 101)
- (n,γ) absorption cross section (MT = 102)
- (n,p) absorption cross section (MT = 103)
- (n,d) absorption cross section (MT = 104)
- (n,t) absorption cross section (MT = 105)
- $(n,^3\text{He})$ absorption cross section (MT = 106)
- (n,α) absorption cross section (MT = 107)
- Neutron production cross section (MT = 1452)
- Flux-weighted CMM diffusion coefficient
- Neutron flux

Diffusion Coefficient Sensitivity Calculations: Cumulative Migration Method

- Developed by Liu in 2016 [1], the Cumulative Migration Method (CMM) allows for highly accurate diffusion coefficient calculations using the concept of “Migration Area”:

$$M^2 = \frac{D}{\Sigma_r} = \frac{1}{6} \bar{r}^2 \qquad R(D_{CMM}) = \frac{\langle M^2 \Sigma_r \phi \rangle}{\langle \phi \rangle}$$

- This method can face challenges when confronted with non-unit cell systems or non-cuboidal reflecting boundaries.

[1] Z. Liu, K. Smith, and B. Forget, “A Cumulative Migration Method for Computing Rigorous Transport Cross Sections and Diffusion Coefficients for LWR Lattices with Monte Carlo,” *Proc. PHYSOR 2016*, Sun Valley, Idaho, May 1–5, 2016.

GPT Calculations in CE TSUNAMI-3D

- The generalized importance function for a response can be expressed as the sum of two terms: the intra-generation effect term and the inter-generational effect term.
 - The **intra-generation** effect describes how much importance a neutron generates after an event occurs.
 - The **inter-generational** effect describes the importance that is generated by the daughter fission neutrons of the original particle.

$$\Gamma^\dagger(\tau_s) = \frac{1}{Q_s} \left\langle \frac{1}{R} \frac{\partial R}{\partial \phi}(r) \phi(\tau_s \rightarrow r) \right\rangle + \frac{\lambda}{Q_s} \left\langle \Gamma^\dagger(r) P(r) \phi(\tau_s \rightarrow r) \right\rangle$$

- CE TSUNAMI-3D uses the **CLUTCH** sensitivity method to calculate the intra-generation term, and an Iterated Fission Probability-based approach to calculate the inter-generational term.
- For more background on this methodology, see:

C. M. Perfetti, B. T. Rearden, "Continuous-Energy Monte Carlo Methods for calculating Generalized Response Sensitivities using TSUNAMI-3D," in *Proc. of the 2014 International Conference on the Physics of Reactors (PHYSOR 2014)*, Kyoto, Japan, September 28 – October 3, 2014.

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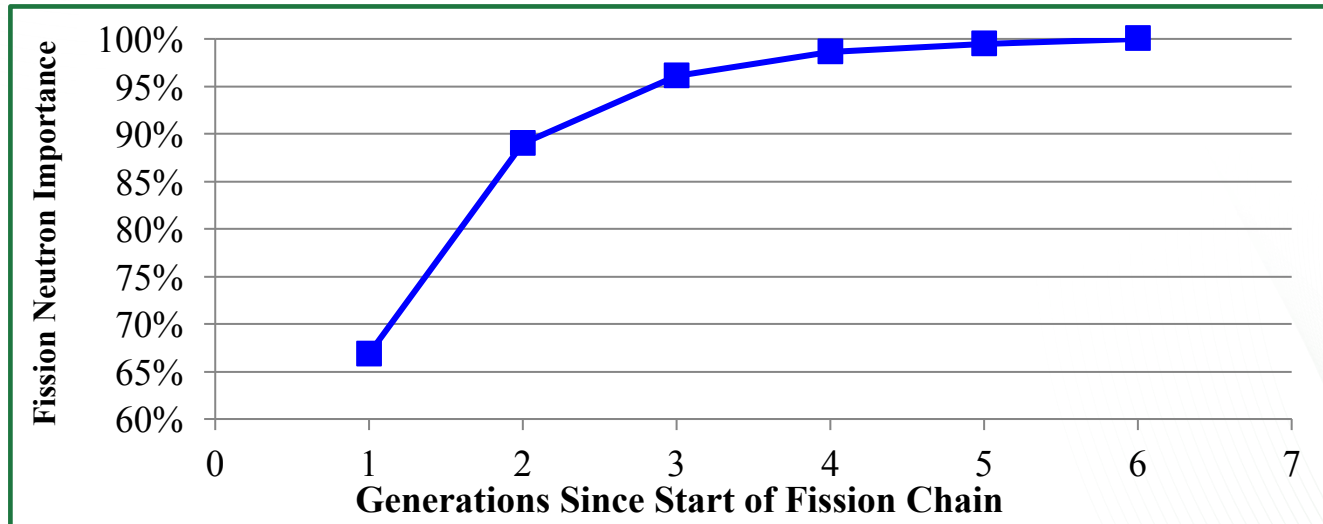
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Inter-generational Importance

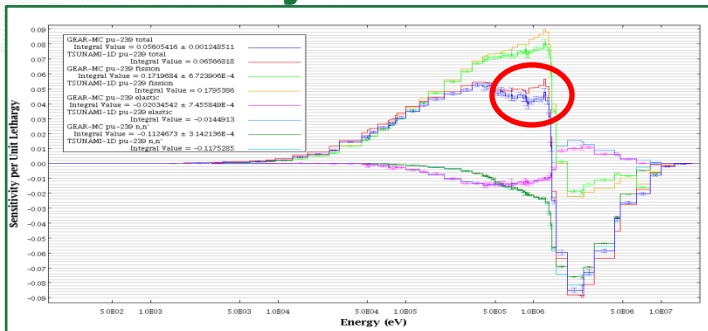
- The **inter-generational** term is calculated by tallying the intra-generational importance generated by neutrons in a fission chain as that importance approaches zero.
 - This term is tallied using the IFP method.
 - The number of “latent” generations used for tallying the inter-generational effect is specified using the **cfp=#** keyword.



GPT Flattop Foil Response Sensitivity Coefficients

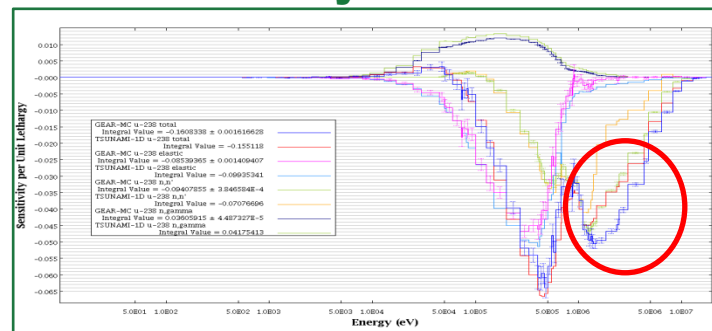
F28/F25 Pu-239

Sensitivity Coefficients



F37/F25 U-238

Sensitivity Coefficients



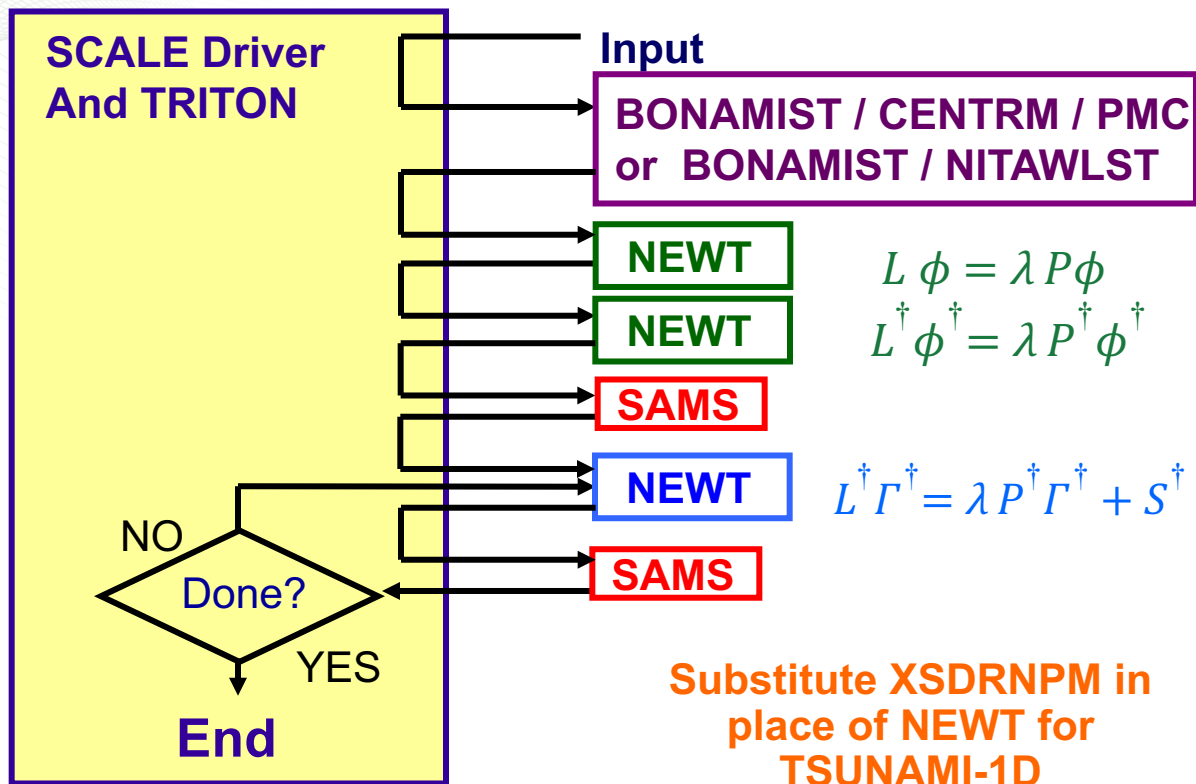
Flattop Total Nuclide Foil Response Sensitivities

Experiment	Response	Isotope	Direct Pert.	TSUNAMI-1D	GEAR-MC
Flattop	F28 / F25	^{238}U	0.8006 ± 0.0533	0.8024 (0.03 σ)	0.7954 ± 0.0018 (-0.10 σ)
		^{239}Pu	0.0528 ± 0.0043	0.0657 (2.99 σ)	0.0561 ± 0.0012 (0.73 σ)
	F37 / F25	^{238}U	-0.1540 ± 0.0102	-0.1551 (-0.11 σ)	-0.1608 ± 0.0016 (-0.66 σ)
		^{239}Pu	0.0543 ± 0.0048	0.0736 (3.99 σ)	0.0489 ± 0.0010 (-1.10 σ)

How does the CE TSUNAMI-3D approach differ from other methods?

- Generalized Perturbation Theory Monte Carlo methods have been developed by Abdel-Khalik et al. for calculating generalized sensitivity coefficients in 3D, continuous-energy Monte Carlo applications, but these methods require performing multiple direct perturbation calculations and can require a large number of runs to calculate generalized sensitivity coefficients.
- This approach differs in that it:
 - Requires no perturbation calculations and no knowledge of nuclear covariance data.
 - Because our approach is not perturbation-based, we can easily calculate energy-dependent sensitivity coefficients for multiple responses to all input nuclear data parameters in one continuous-energy Monte Carlo transport calculation.
 - The deterministic, sensitivity-based TSUNAMI-1D and TSUNAMI-2D GPT methods require at least one transport calculation per generalized response.

TSUNAMI-1D/2D GPT Sequences



Resonance cross-section processing
 (repeated for all cells)

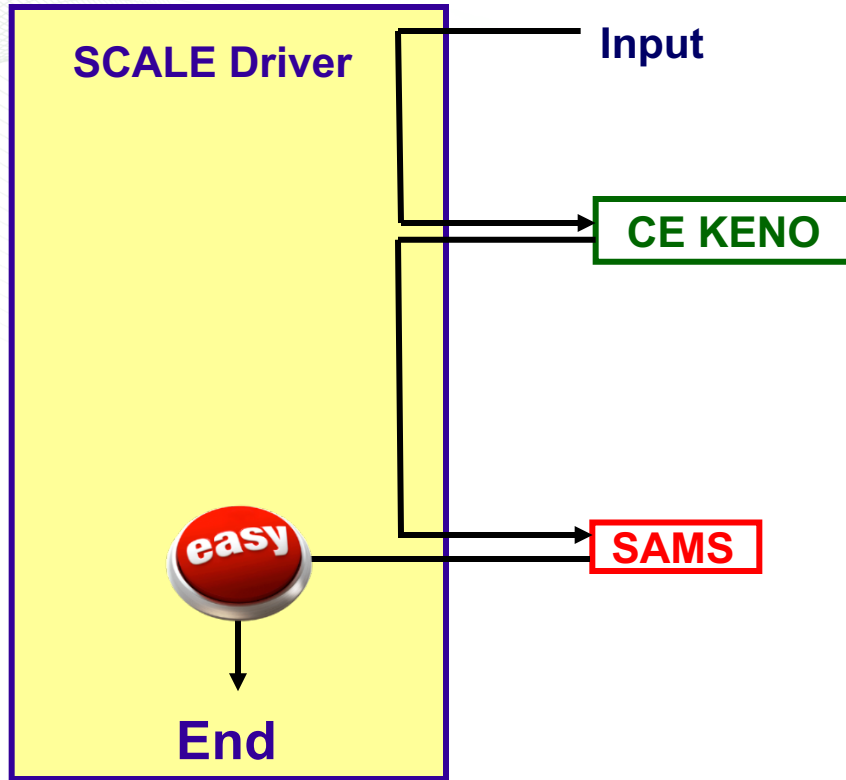
2D discrete ordinates
 2D discrete ordinates
 adjoint calculation

S/U calculation for k_{eff}

2D discrete ordinates
 inhomogeneous adjoint
 calculation for each response

S/U calculation for a user-
 defined response

CE TSUNAMI-3D GPT Sequence



3D Monte Carlo

$$L \phi = \lambda P \phi$$

$$L^\dagger \phi^\dagger = \lambda P^\dagger \phi^\dagger$$

$$L^\dagger \Gamma^\dagger = \lambda P^\dagger \Gamma^\dagger + S^\dagger$$

S/U calculation for k_{eff}
and user-defined
responses

Definitions Block

- Used to define reaction rates, or responses, for GPT sensitivities.
- **mixture=#** is used to define the material for the response.
 - **multimix=#1 #2 #3 end** is used to define responses containing multiple materials.
- **ehigh=#1** and **elow=#2** will create an energy window for this response.

```
read definitions
response 5
    nuclide=92235
    reaction=fission
    mixture=10 micro
    ehigh=0.625
end response
response 6
    unity mixture=10
end response
end definitions
```

Definitions Block

- **reaction=#** keyword is used to define the reaction of interest.
 - Omitting this keyword and entering “unity” will result in a flux response.
 - Reactions available in CE TSUNAMI-3D:
 - mt=1 (total XS)
 - mt=18 (fission)
 - mt=102 (n,gamma)
 - mt=452 (nu-bar)
- **nuclide=ZZAAA** will tally the response for only one nuclide.

```
read definitions
response 5
    nuclide=92235
    reaction=fission
    mixture=10 micro
    ehigh=0.625
end response
response 6
    unity mixture=10
end response
end definitions
```

SystemResponses Block

```
read systemresponses
  ratio 1 title='U235-fis'
    numer 5 end
    denom 6 end
  end ratio
end systemresponses
```

- Each response must have its own **ratio #** and **end ratio** input lines.
- The **numer** keyword is used to specify which Definition is in the response numerator.
- The **denom** keyword is used to specify which Definition is in the response denominator.

Summary

- The CE TSUNAMI-3D code within the SCALE 6.2 code package offers a variety of approaches for calculating sensitivity coefficients for both eigenvalue and GPT responses.
- The GPT TSUNAMI capabilities expand the range of applicability for SCALE S/U analyses.

Questions?

Please contact:

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